October 15

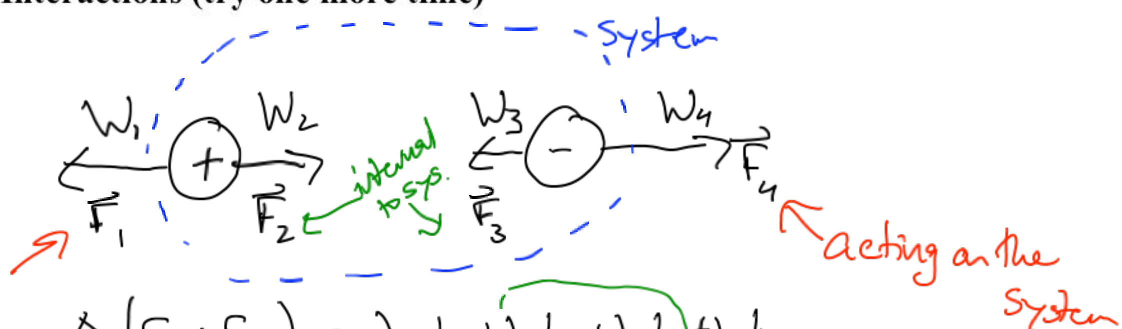
Get Whiteboards

I will teach 0175 next semester
WF from 2:00-3:40 and M 2:00-2:50

The class is listed with course id number 33982

It should be open to everyone, so if you want a spot, I suggest registering as early as possible…

Interactions (try one more time)



$$\Delta(E_p + E_e) = W_1 + W_2 + W_3 + W_4$$

internal to system

$$\Delta(E_p + E_e) - W_2 - W_3 = W_1 + W_4$$

$$\Delta U = -(W_2 + W_3)$$ change in potential energy

$$\Delta(E_e + E_p + U_{ep}) = W_1 + W_4$$

$$\Delta(E_{sys} + U_{sys}) = W_{surroundings}$$

Relating force to potential energy

$$F_r = -\frac{dU}{dr}$$

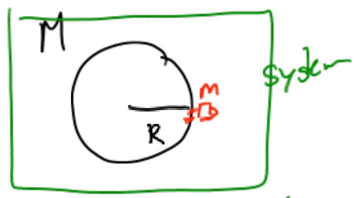
Gravitational potential energy

$$F = -\frac{Gm_1m_2}{r^2} \qquad \text{What is } U?$$

$$dU = -F\,dr = +\frac{Gm_1m_2}{r^2}dr$$

$$U = \int dU = \int \frac{Gm_1m_2}{r^2}dr = -\frac{Gm_1m_2}{r}$$

Ponderable: It's rocket science!



no external forces $\Rightarrow$ no work done on system

If sys was rocket

$$\Delta E = W$$

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \int \vec{F} \cdot d\vec{r}$$

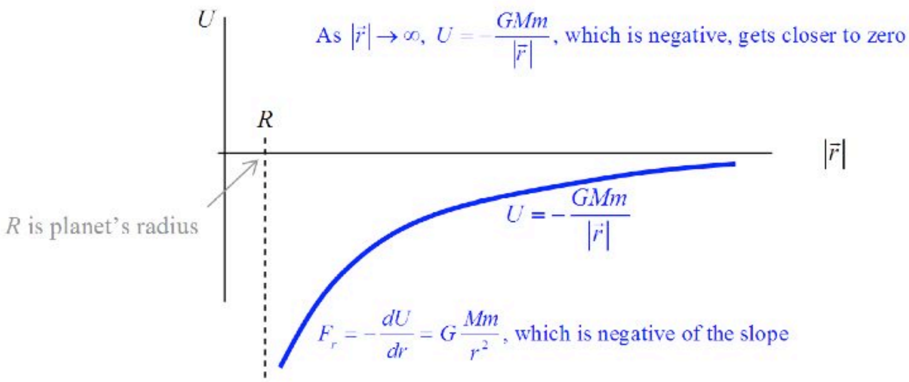
$$E_f = E_i + W = E_i + 0$$

$$\left(\cancel{Mc^2} + \cancel{K_{M,f}} + \cancel{mc^2} + K_{m,f} + U_f\right) = \left(\cancel{Mc^2} + \cancel{K_{M,i}} + \cancel{mc^2} + K_{m,i} + U_i\right)$$

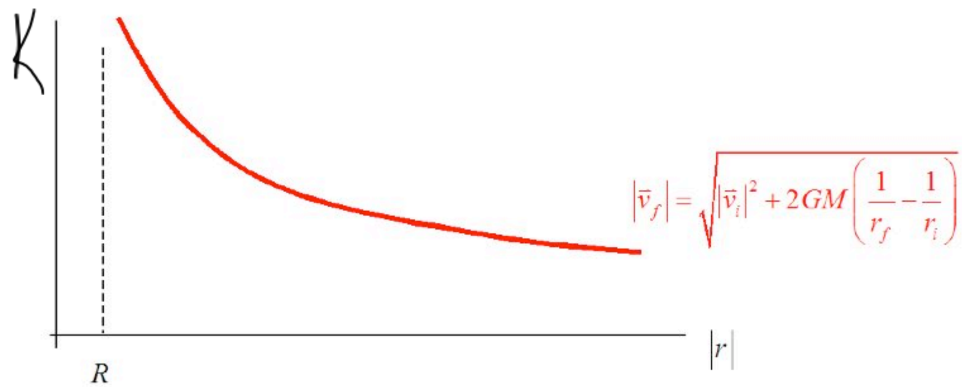
$$K_{m,f} + U_f = K_{m,i} + U_i$$

$$\frac{1}{2} \cancel{m} v_f^2 - \frac{GM\cancel{m}}{r_f} = \frac{1}{2} \cancel{m} v_i^2 - \frac{G\cancel{m}M}{r_i}$$

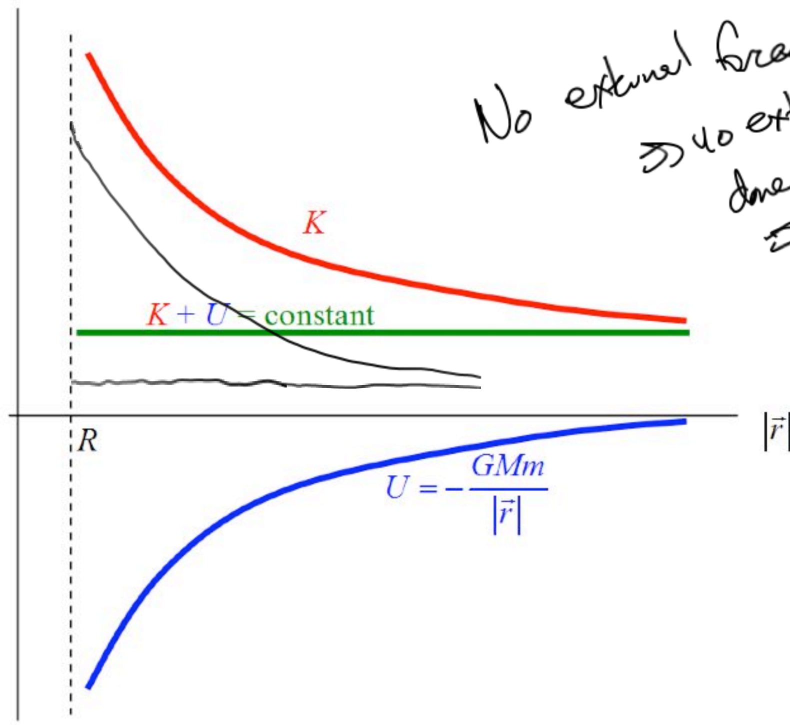
$$|\vec{v}_f| = \sqrt{v_i^2 + 2GM\left(\frac{1}{r_f} - \frac{1}{r_i}\right)}$$


U
As $|\vec{r}| \to \infty$, $U = -\frac{GMm}{|\vec{r}|}$, which is negative, gets closer to zero
R
$|\vec{r}|$
R is planet's radius
$U = -\frac{GMm}{|\vec{r}|}$
$F_r = -\frac{dU}{dr} = G\frac{Mm}{r^2}$, which is negative of the slope

$$F_r = -\frac{dU}{dr}$$



K
R
$|r|$
$|\bar{v}_f| = \sqrt{|\bar{v}_i|^2 + 2GM\left(\frac{1}{r_f} - \frac{1}{r_i}\right)}$

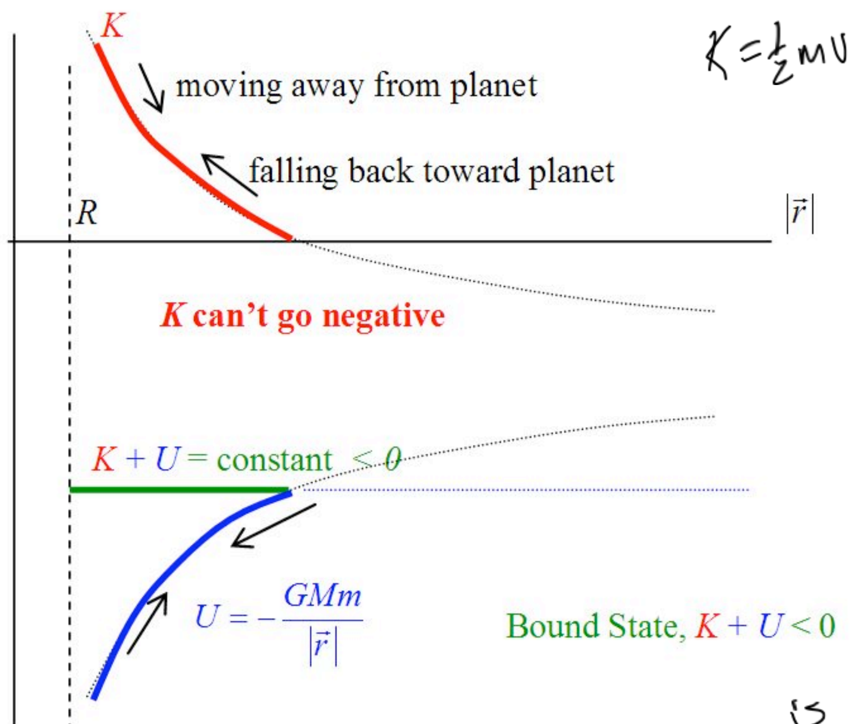


No external forces

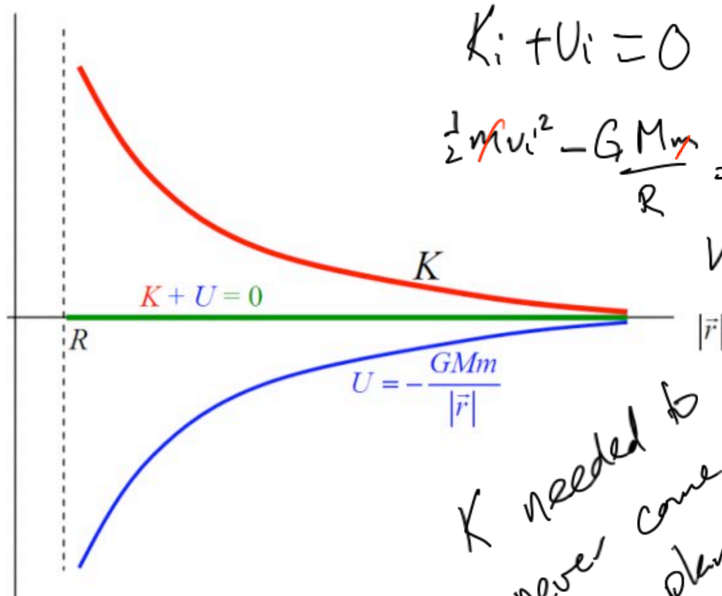
$\Rightarrow$ no external work done

$\Rightarrow E_f = E_i$

$K = \frac{1}{2}mv^2 \geq 0$



If V_i is small enough
$K + U < 0$



$$K_i + U_i = 0$$

$$\frac{1}{2}\cancel{m}v_i^2 - \frac{GM\cancel{m}}{R} = 0$$

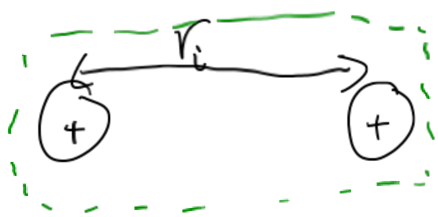
$$v_i = \sqrt{\frac{2GM}{R}}$$

K needed to never come back to the planet

$\Rightarrow \frac{1}{2}mv_i^2$ is minimum initial K

v_i is called escape velocity

2 Protons?



hold fix and let them go

$$|\vec{F}_{elec}| = \frac{1}{4\pi\epsilon_0}\frac{|Qq|}{|\vec{r}|^2}$$

$$U = +\frac{1}{4\pi\epsilon_0}\frac{Qq}{r}$$

$$F_{grav} = -\frac{G m_1 m_2}{|\vec{r}|^2}$$

$$\downarrow$$

$$U = -\frac{G m_1 m_2}{r}$$

$$E_f = E_i + W = E_i + 0$$

$$\cancel{mc^2} + K_{1,f} + \cancel{mc^2} + K_{2f} + U_f = \cancel{mc^2} + K_{1,i} + \cancel{mc^2} + K_{2,i} + U_i$$

$$K_{1f} + K_{2f} + U_f = K_{1i} + K_{2i} + U_i$$

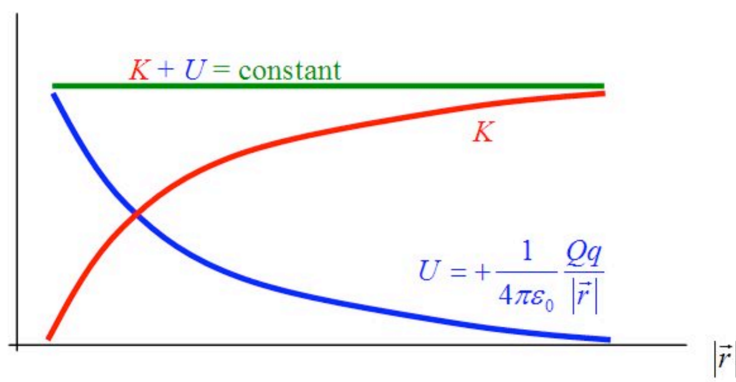
released from rest

$$K_{1i} = K_{2i} = 0$$

$$K_{1f} + K_{2f} + U_f = U_i$$

drop to zero

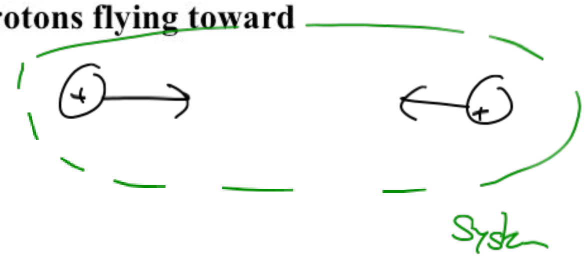
$$K_{1f} + K_{2f} = U_i$$



$K + U =$ constant
K
$U = +\frac{1}{4\pi\varepsilon_0}\frac{Qq}{|\vec{r}|}$
$|\vec{r}|$

Discussion: Two widely separated protons flying toward each other

$$E_f = E_i + \overset{0}{W} = E_i$$



$$\cancel{m_1c^2} + \cancel{m_2c^2} + K_{1f} + K_{2f} + U_f = \cancel{m_1c^2} + \cancel{m_2c^2} + K_{1i} + K_{2i} + U_i$$

0 since start far apart

look at final state where they reach closest approach

$$K_{1f} = K_{2f} = 0$$

$$U_f = K_{1i} + K_{2i} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r}$$

Ponderable: Typical ionization energy

System: outer electron and atom.
Initial state: electron in orbit r = 1e-10 m
Final state: electron at infinity at rest.

Assume initial kinetic energy is small. Single ionization leaves ion with +e charge.

$$E_f = E_i + W$$

$$\cancel{m_{ion}c^2} + \cancel{K_{ion,f}} + \cancel{m_e c^2} + \cancel{K_{e,f}} + \cancel{U_f} = \cancel{m_{ion}c^2} + \cancel{K_{ion,i}} + \cancel{m_e c^2} + \cancel{K_{e,i}} + U_i + W$$

$$0 = U_i + W$$

$$\Rightarrow W = -U_i = -\frac{1}{4\pi\epsilon_0}\frac{q_{ion}q_e}{r_i}$$

$$= +\left(9\times10^9 \frac{N\cdot m^2}{C^2}\right)\frac{(1.6\times10^{-19}\,C)^2}{10^{-10}\,m} = 2.3\times10^{-18}\,J$$

$$= 14\,eV$$

Discussion: Review of potential energy and forces

$$F_s = k_s s \qquad U_s = \tfrac{1}{2} k_s s^2$$

$$F_g = G\frac{m_1 m_2}{r^2} \qquad U_g = -G\frac{m_1 m_2}{r}$$

$$F_{el} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1 q_2}{r^2} \qquad U_{el} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q_1 q_2}{r}$$

Ponderable: 5.P.81 Nuclear Fission (page 219)

Uranium-235 fissions when it absorbs a slow-moving neutron. The two fission fragments can be almost any two nuclei whose charges Q_1 and Q_2 add up to $92e$ (where e is the charge on a proton, e = 1.6e-19 coulomb), and whose nucleons add up to 236 protons and neutrons (U-236; U-235 plus a neutron). One of the possible fission modes involves nearly equal fragments, palladium nuclei (Pd) each with electric charge $Q_1 = Q_2 = 46e$. The rest masses of the two palladium nuclei add up to less than the rest mass of the original nucleus. (In addition to the two main fission fragments there are typically one or more free neutrons in the final state; in your analysis make the simplifying assumption that there are no free neutrons, just two palladium nuclei.)

The rest mass of the U-236 nucleus (formed from U-235 plus a neutron) is 235.996 u (unified atomic mass units), and the rest mass of each of the two Pd-118 nuclei is 117.894 u, where 1 u = 1.66e-27 kg (approximately the mass of one nucleon).

Keep at least 6 significant figures, because the calculations involve subtracting large numbers from each other, leaving a small difference. Don't plug in numbers until the end.

(a) Calculate the final speed v, when the palladium nuclei have moved very far apart (due to their mutual electric repulsion).

(b) Using energy considerations, calculate the distance between centers of the palladium nuclei just after fission, when they are momentarily at rest.

(c) A proton or neutron has a radius of roughly $1\text{x}10^{-15}$ m, and a nucleus is a tightly packed collection of nucleons. Experiments show that the radius of a nucleus containing N nucleons is pproximately $(1.3 \text{ x } 10^{-15} \text{ m}) \text{ x } N^{1/3}$. What is the approximate radius of a palladium nucleus? Draw a sketch of the palladium nuclei in part (b), and label the distances you calculated in parts (b) and (c).

d) As a check, and for further practice, find the distance between centers of the palladium nuclei just after fission, when they are momentarily at rest using state 2 as your initial state and state 3 as your final state. This should agree with your results in part (b).

3 possible states

GATHER:
$M_{U\text{-}236}$ = 235.996 u; $M_{Pd\text{-}118}$= 117.894 u; 1 u = 1.66e-27 kg;
$Q_{Pd\text{-}118}$ = 46 *e*; *e* = 1.6e-19 coulomb
It is ok to use the nonrelativistic formulas, but must check that the calculated *v* is indeed small compared to *c*. (The large kinetic energies of these palladium nuclei are eventually dissipated into thermal energy of the surrounding material. In a nuclear reactor this hot material boils water and drives an electric generator.)

ORGANIZE:
Chose a system: all the particles (no external work)
Identify initial and final states (there are three choices):

1. The U-236 nucleus before it fissions
2. Just after fission when the two palladium nuclei are close together and momentarily at rest.
3. The palladium nuclei are very far away from each other, traveling at high speeds

Initial State: State 1
Final State: State 3
Draw appropriate diagrams
Identify appropriate Fundamental Principle: Energy Principle

ANALYZE:

The analysis can be thought of as a diamond.

- Write a compact statement of the energy principle for your system and choice of initial and final states.
- Expand to include all the possible energy terms.
- Rewrite with appropriate subscripts for the situation.
- Contract by evaluating specific terms.
- Solve for the unknown quantity of interest.

$$E_f = E_i + W_{\text{ext}}$$

$$\left(m_{1f}c^2 + K_{1f}\right) + \left(m_{2f}c^2 + K_{2f}\right) + \ldots + U_{12f} + \ldots = \left(m_{1i}c^2 + K_{1i}\right) + \left(m_{2i}c^2 + K_{2i}\right) + \ldots + U_{12i} + \ldots + W_{\text{ext}}$$

Rewrite with appropriate subscripts for the particular situation.

Cross out any terms that are zero;

write specific potential energy terms.

Solve for unknown.

Plug in numbers.

$2m_{Pd}c^2$ |

LEARN: Do the units make sense? Did the speed of each palladium nucleus turn out to be small enough that $\frac{1}{2}mv^2$ or $\frac{p^2}{2m}$ was an adequate approximation for the kinetic energy of one of the palladium nuclei? Is the final speed a high speed? (High speed goes with lots of heating of the metal, which can run electric generators.)

$$2 m_{Pd} c^2 + 2\left(\tfrac{1}{2} m_{Pd} v^2\right) + \cancel{U_A} = m_U c^2$$

$$m_{Pd} v^2 = m_U c^2 - 2 m_{Pd} c^2$$

$$v = \sqrt{\frac{m_U c^2 - 2 m_{Pd} c^2}{m_{Pd}}}$$

$$m_U c^2 = 2 m_{Pd} c^2 + \frac{1}{4\pi\varepsilon_0} \frac{Q^2}{r}$$